Fluid Flow in a Micro Hydro System

A Sustainable Solution for Narukunibua, Fiji

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Victoria Johnson and Jenna Wilson
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Abstract

Our goal is to implement a micro hydro system in Narukunibua, a rural village in Fiji, as part of our work with the Sustainable Energy Initiative. We examined a number of different turbine and penstock options to include in our system, narrowing down our options through research and basic fluid flow calculations. After performing these tasks, we determined that an impulse turbine such as the Pelton wheel, Banki-Mitchell turbine, or Turgo turbine would best fit our conditions. In addition, we recommend using a high head, low flow situation in order to reduce frictional losses and increase efficiency.

Introduction

The Sustainable Energy Initiative is a campus club which tries to raise awareness of and implement sustainable technologies locally and internationally. As members, our primary project is to implement a sustainable energy system in the rural Fijian village of Narukunibua, replacing their current use of a diesel engine. One of the primary sources the club is considering is micro hydro power, which is small scale hydroelectric system. The area is ideal for this energy source, as it is very mountainous, has heavy rainfall, and is located next to a large river. For our special project, we will attempt to find the optimum design of fluid flow for the system while adhering to our given conditions.

We will examine a number of possible micro hydro systems in order to discover which one best fits the needs of the village and is best suited to their surroundings. After visiting the village and personally surveying the environment, as well as collecting the views of the villagers, we will be able to choose the most appropriate system to implement. We will consider five different turbine-types, as well as the corresponding penstock for water delivery. These are: the Pelton wheel, the Banki-Mitchell turbine, the Turgo turbine, the Francis turbine, and the Kaplan turbine.

Description of the Problem – Turbine Types

Pelton Wheel:

The Pelton wheel is currently the most widely-used turbine in small micro hydro systems due to its acceptance of flow variation and low flow rate requirement. It falls under the category of “impulse turbines,” meaning that it converts the potential energy of the incoming water into kinetic energy using a nozzle to create a high-speed jet. This jet is applied to a series of buckets, turning the wheel and creating mechanical energy. Using a generator, this mechanical energy can be converted to electrical energy.

The advantages of the Pelton wheel include:
- High acceptance of flow variation (could operate in different seasons and while unmanned)
- Does not need to be encased
- Can deliver significant power with low flow rates
- Relative ease of construction, where the different components are able to be made separately and then assembled
- Relatively easy maintenance since it can be taken apart

The disadvantages of the Pelton wheel include:
- Need a high pressure head in order to produce power (larger than 60 meters)

- Larger amount of penstock needed due to long distance
- Needs well-filtered water to prevent nozzle blockage

Banki-Mitchell Turbine:

The Banki-Mitchell turbine is also an impulse turbine, but unlike the Pelton wheel, it uses a cross-flow of water to generate power. In a cross-flow turbine, the water moves through the turbine transversely, or across the turbine blades. As with a waterwheel, the water is admitted at the turbine's edge and leaves on the opposite side after traveling through the runner.

The advantages of the Banki-Mitchell turbine include:
- Can operate between a large range of head heights (5 – 200 m)
- Simple design is inexpensive and easy to repair
- High acceptance of flow variation (could operate in different seasons and while unmanned)
- More reliable that other turbine types
- Has a self-cleaning mechanism in which the water helps clean the runner of small debris
- Does not need to be encased

The disadvantages of the Banki-Mitchell turbine include:
- Low efficiencies in comparison to other turbines
- Sharp blades may make turbine unsafe
- Needs well-filtered water to prevent nozzle blockage

Turgo Turbine:

The Turgo turbine is another impulse type turbine, similar to the Pelton and Banki-Mitchell turbines. A high speed jet of water is applied to the blades of the turbine which then deflects and reverses the flow. This impulse causes the turbine runner to spin, passing energy to the shaft of the turbine.

The advantages of the Turgo turbine include:
- High acceptance of flow variation
- Works well with long penstock situations
- Inexpensive to manufacture
- Does not need to be encased

The disadvantages of the Turgo turbine include:
- Low efficiencies in comparison to other turbines
- Need a high pressure head in order to produce power (50 – 250m)
- Larger amount of penstock needed due to long distance
- Needs well-filtered water to prevent nozzle blockage

Francis Turbine:

The Francis turbine is a reaction turbine that functions by water entering the turbine radially by spiral case and exiting axially. The case directs the water tangentially to the runner which causes the runner to spin. The water’s pressure and velocity is converted into kinetic energy of the turbine. The Francis turbine has adjustable guide vanes before the runner to accommodate a range of water flow conditions and can be oriented vertically or horizontally

The advantages of the Francis turbine include:
- High efficiency
- Can operate between a large range of head heights (25 – 350m)
- Medium acceptance of flow variation

The disadvantages of the Francis turbine include:
- Low acceptance of head variation
- Must be fully immersed in water and enclosed in a pressure casing (difficult to repair and reproduce)
- Usually used only in large hydroelectric systems
- Difficult to maintain/clean
- Guide vanes must be adjusted based on current flow rate, so it needs to be manned

Kaplan Turbine:

Kaplan turbines are axial flow reaction turbines that are generally used in low heads. It has adjustable runner blades and may have adjustable guide-vanes. If it has both, it is called a double regulated Kaplan turbine, otherwise it is called a single regulated Kaplan turbine.

The advantages of the Kaplan turbine include:
- Good adaption to varying flow rates
- Can be used in a low head situation (2 – 40m), leading to lower cost of penstock/piping

The disadvantages of the Kaplan turbine include:
- Single regulated does not easily adapt to varying head
- Requires a high flow rate
- Must be fully immersed in water and enclosed in a pressure casing (difficult to repair and reproduce)
- Runner blades/guide vanes must be adjusted based on current flow rate, so it needs to be manned
- Expensive to design, manufacture, and install

Illustration

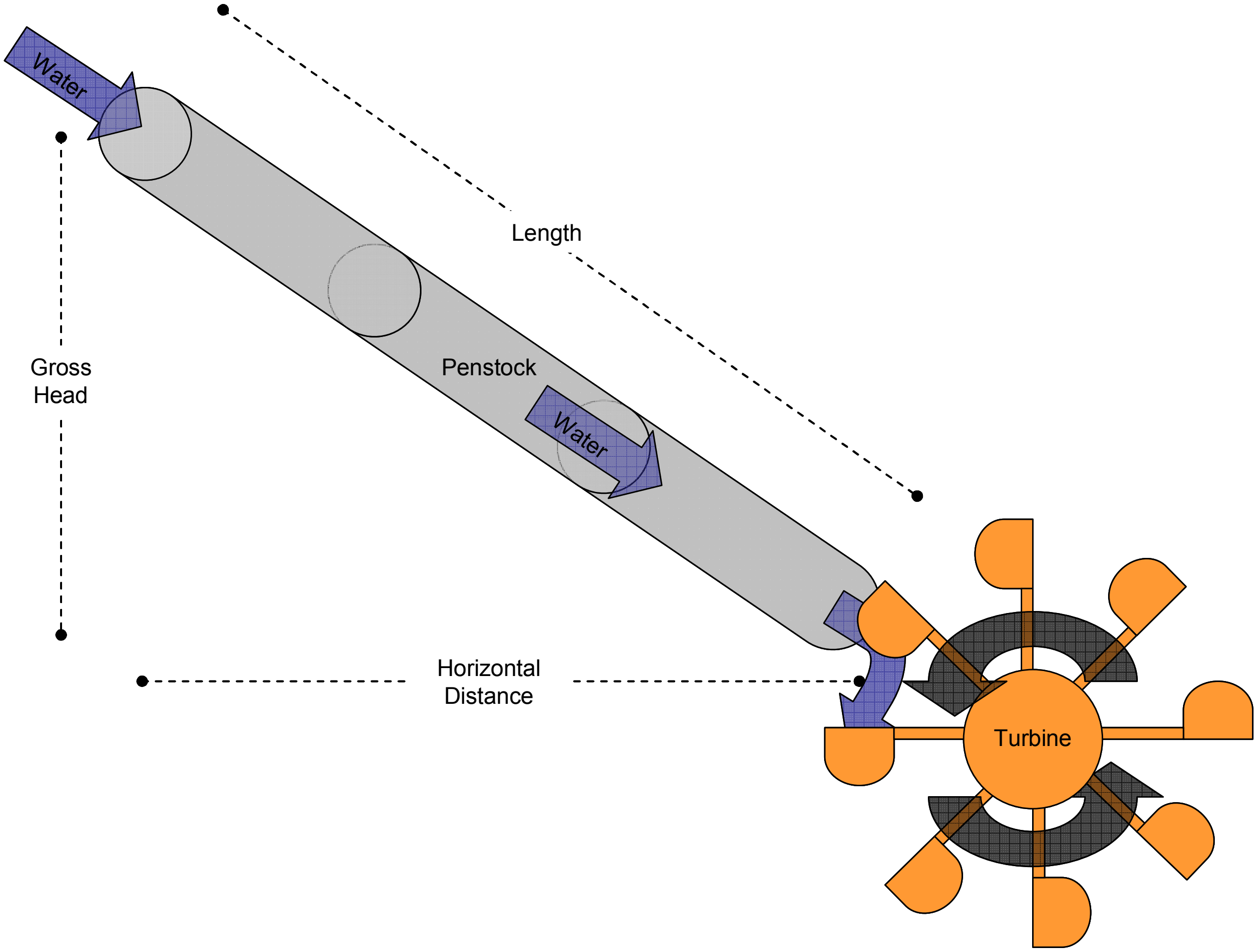


Figure 1. Schematic of a basic micro hydro system

Mathematical Model & Solution

The power available from a stream of water is determined by:

$$P = \eta \times \rho \times g \times H_{net} \times Q$$

where:

η = efficiency of turbine
ρ = density of water [kg/m^3]
g = gravitational constant [m/s^2]
H_{net} = net head [m]
Q = volumetric flow rate [m^3/s]

Using the optimum conditions of each turbine, the power output based on volumetric flow rate and net head was calculated. In these cases, we used the maximum efficiency of each turbine detailed by the European Small Hydropower Association. Once these values were calculated, the combinations which produced between 10 – 20 kW were highlighted, as this is the target range of energy production for the village. These data can be seen in *Tables 2 – 6*, below.

The efficiencies used are shown in *Table 1* below:

Table 1. Turbine Efficiencies

Turbine	η
Pelton	0.90
Banki-Mitchell	0.87
Turgo	0.85
Francis	0.90
Kaplan	0.90

Table 2. Estimated Power Output (kW) from a Pelton Wheel

	Net Head											
Flow Rate (m^3/s)	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**
0.0005	0.04	0.09	0.13	0.18	0.22	0.26	0.31	0.35	0.40	0.44	0.49	0.53
0.001	0.09	0.18	0.26	0.35	0.44	0.53	0.62	0.71	0.79	0.88	0.97	1.06
0.002	0.18	0.35	0.53	0.71	0.88	1.06	1.24	1.41	1.59	1.77	1.94	2.12
0.003	0.26	0.53	0.79	1.06	1.32	1.59	1.85	2.12	2.38	2.65	2.91	3.18
0.004	0.35	0.71	1.06	1.41	1.77	2.12	2.47	2.83	3.18	3.53	3.88	4.24
0.006	0.53	1.06	1.59	2.12	2.65	3.18	3.71	4.24	4.77	5.30	5.83	6.36
0.008	0.71	1.41	2.12	2.83	3.53	4.24	4.94	5.65	6.36	7.06	7.77	8.48
0.010	0.88	1.77	2.65	3.53	4.41	5.30	6.18	7.06	7.95	8.83	9.71	10.59
0.012	1.06	2.12	3.18	4.24	5.30	6.36	7.42	8.48	9.54	10.59	11.65	12.71
0.014	1.24	2.47	3.71	4.94	6.18	7.42	8.65	9.89	11.12	12.36	13.60	**14.83**
0.016	1.41	2.83	4.24	5.65	7.06	8.48	9.89	11.30	12.71	14.13	15.54	16.95
0.018	1.59	3.18	4.77	6.36	7.95	9.54	11.12	12.71	14.30	15.89	17.48	19.07
0.020	1.77	3.53	5.30	7.06	8.83	10.59	12.36	14.13	15.89	17.66	19.42	21.19
0.025	2.21	4.41	6.62	8.83	11.04	13.24	15.45	17.66	19.87	22.07	24.28	26.49
0.030	2.65	5.30	7.95	10.59	13.24	**15.89**	18.54	21.19	23.84	26.49	29.14	31.78
0.035	3.09	6.18	9.27	12.36	15.45	18.54	21.63	24.72	27.81	30.90	33.99	37.08
0.040	3.53	7.06	10.59	14.13	17.66	21.19	24.72	28.25	31.78	35.32	38.85	42.38
0.045	3.97	7.95	11.92	15.89	19.87	23.84	27.81	31.78	35.76	39.73	43.70	47.68
0.050	4.41	8.83	13.24	17.66	22.07	26.49	30.90	35.32	39.73	44.15	48.56	52.97
0.075	6.62	13.24	19.87	26.49	33.11	39.73	46.35	52.97	59.60	66.22	72.84	79.46
0.100	8.83	17.66	26.49	35.32	44.15	52.97	61.80	70.63	79.46	88.29	97.12	105.95

*Pelton wheel requires net head greater than 60 m

Table 3. Estimated Power Output (kW) from a Banki-Mitchell Turbine

	Net Head											
Flow Rate (m^3/s)	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**
0.0005	0.04	0.09	0.13	0.17	0.21	0.26	0.30	0.34	0.38	0.43	0.47	0.51
0.001	0.09	0.17	0.26	0.34	0.43	0.51	0.60	0.68	0.77	0.85	0.94	1.02
0.002	0.17	0.34	0.51	0.68	0.85	1.02	1.19	1.37	1.54	1.71	1.88	2.05
0.003	0.26	0.51	0.77	1.02	1.28	1.54	1.79	2.05	2.30	2.56	2.82	3.07
0.004	0.34	0.68	1.02	1.37	1.71	2.05	2.39	2.73	3.07	3.41	3.76	4.10
0.006	0.51	1.02	1.54	2.05	2.56	3.07	3.58	4.10	4.61	5.12	5.63	6.14
0.008	0.68	1.37	2.05	2.73	3.41	4.10	4.78	5.46	6.14	6.83	7.51	8.19
0.010	0.85	1.71	2.56	3.41	4.27	5.12	5.97	6.83	7.68	8.53	9.39	10.24
0.012	1.02	2.05	3.07	4.10	5.12	6.14	7.17	8.19	9.22	10.24	11.27	12.29
0.014	1.19	2.39	3.58	4.78	5.97	7.17	8.36	9.56	10.75	11.95	13.14	14.34
0.016	1.37	2.73	4.10	5.46	6.83	8.19	9.56	10.92	12.29	13.66	15.02	16.39
0.018	1.54	3.07	4.61	6.14	7.68	9.22	10.75	12.29	13.83	15.36	16.90	18.43
0.020	1.71	3.41	5.12	6.83	8.53	10.24	11.95	13.66	15.36	17.07	18.78	20.48
0.025	2.13	4.27	6.40	8.53	10.67	12.80	**14.94**	17.07	19.20	21.34	23.47	25.60
0.030	2.56	5.12	7.68	10.24	12.80	15.36	17.92	20.48	23.04	25.60	28.16	30.72
0.035	2.99	5.97	8.96	11.95	14.94	17.92	20.91	23.90	26.88	29.87	32.86	35.85
0.040	3.41	6.83	10.24	13.66	17.07	20.48	23.90	27.31	30.72	34.14	37.55	40.97
0.045	3.84	7.68	11.52	15.36	19.20	23.04	26.88	30.72	34.57	38.41	42.25	46.09
0.050	4.27	8.53	12.80	17.07	21.34	25.60	29.87	34.14	38.41	42.67	46.94	51.21
0.060	5.12	10.24	**15.36**	20.48	25.60	30.72	35.85	40.97	46.09	51.21	56.33	61.45
0.070	5.97	11.95	17.92	23.90	29.87	35.85	41.82	47.79	53.77	59.74	65.72	71.69
0.075	6.40	12.80	19.20	25.60	32.01	38.41	44.81	51.21	57.61	64.01	70.41	76.81
0.100	8.53	17.07	25.60	34.14	42.67	51.21	59.74	68.28	76.81	85.35	93.88	102.42

Table 4. Estimated Power Output (kW) from a Turgo Turbine

	Net Head											
Flow Rate (m^3/s)	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**
0.0005	0.04	0.08	0.13	0.17	0.21	0.25	0.29	0.33	0.38	0.42	0.46	0.50
0.001	0.08	0.17	0.25	0.33	0.42	0.50	0.58	0.67	0.75	0.83	0.92	1.00
0.002	0.17	0.33	0.50	0.67	0.83	1.00	1.17	1.33	1.50	1.67	1.83	2.00
0.003	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
0.004	0.33	0.67	1.00	1.33	1.67	2.00	2.33	2.67	3.00	3.34	3.67	4.00
0.006	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00
0.008	0.67	1.33	2.00	2.67	3.34	4.00	4.67	5.34	6.00	6.67	7.34	8.00
0.010	0.83	1.67	2.50	3.34	4.17	5.00	5.84	6.67	7.50	8.34	9.17	10.01
0.012	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.01	10.01	11.01	12.01
0.014	1.17	2.33	3.50	4.67	5.84	7.00	8.17	9.34	10.51	11.67	12.84	14.01
0.016	1.33	2.67	4.00	5.34	6.67	8.00	9.34	10.67	12.01	13.34	14.68	16.01
0.018	1.50	3.00	4.50	6.00	7.50	9.01	10.51	12.01	13.51	**15.01**	16.51	18.01
0.020	1.67	3.34	5.00	6.67	8.34	10.01	11.67	13.34	15.01	16.68	18.34	20.01
0.025	2.08	4.17	6.25	8.34	10.42	12.51	14.59	16.68	18.76	20.85	22.93	25.02
0.030	2.50	5.00	7.50	10.01	12.51	15.01	17.51	20.01	22.51	25.02	27.52	30.02
0.035	2.92	5.84	8.76	11.67	**14.59**	17.51	20.43	23.35	26.27	29.18	32.10	35.02
0.040	3.34	6.67	10.01	13.34	16.68	20.01	23.35	26.68	30.02	33.35	36.69	40.02
0.045	3.75	7.50	11.26	15.01	18.76	22.51	26.27	30.02	33.77	37.52	41.28	45.03
0.050	4.17	8.34	12.51	16.68	20.85	25.02	29.18	33.35	37.52	41.69	45.86	50.03
0.060	5.00	10.01	15.01	20.01	25.02	30.02	35.02	40.02	45.03	50.03	55.03	60.04
0.070	5.84	11.67	17.51	23.35	29.18	35.02	40.86	46.70	52.53	58.37	64.21	70.04
0.075	6.25	12.51	18.76	25.02	31.27	37.52	43.78	50.03	56.28	62.54	68.79	75.05
0.100	8.34	16.68	25.02	33.35	41.69	50.03	58.37	66.71	75.05	83.39	91.72	100.06

*Turgo turbine requires head greater than 50 m

Table 5. Estimated Power Output (kW) from a Francis Turbine

	Net Head											
Flow Rate (m^3/s)	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**
0.0005	0.04	0.09	0.13	0.18	0.22	0.26	0.31	0.35	0.40	0.44	0.49	0.53
0.001	0.09	0.18	0.26	0.35	0.44	0.53	0.62	0.71	0.79	0.88	0.97	1.06
0.002	0.18	0.35	0.53	0.71	0.88	1.06	1.24	1.41	1.59	1.77	1.94	2.12
0.003	0.26	0.53	0.79	1.06	1.32	1.59	1.85	2.12	2.38	2.65	2.91	3.18
0.004	0.35	0.71	1.06	1.41	1.77	2.12	2.47	2.83	3.18	3.53	3.88	4.24
0.006	0.53	1.06	1.59	2.12	2.65	3.18	3.71	4.24	4.77	5.30	5.83	6.36
0.008	0.71	1.41	2.12	2.83	3.53	4.24	4.94	5.65	6.36	7.06	7.77	8.48
0.010	0.88	1.77	2.65	3.53	4.41	5.30	6.18	7.06	7.95	8.83	9.71	10.59
0.012	1.06	2.12	3.18	4.24	5.30	6.36	7.42	8.48	9.54	10.59	11.65	12.71
0.014	1.24	2.47	3.71	4.94	6.18	7.42	8.65	9.89	11.12	12.36	13.60	14.83
0.016	1.41	2.83	4.24	5.65	7.06	8.48	9.89	11.30	12.71	14.13	15.54	16.95
0.018	1.59	3.18	4.77	6.36	7.95	9.54	11.12	12.71	14.30	15.89	17.48	19.07
0.020	1.77	3.53	5.30	7.06	8.83	10.59	12.36	14.13	15.89	17.66	19.42	21.19
0.025	2.21	4.41	6.62	8.83	11.04	13.24	15.45	17.66	19.87	22.07	24.28	26.49
0.030	2.65	5.30	7.95	10.59	13.24	15.89	18.54	21.19	23.84	26.49	29.14	31.78
0.035	3.09	6.18	9.27	12.36	15.45	18.54	21.63	24.72	27.81	30.90	33.99	37.08
0.040	3.53	7.06	10.59	14.13	17.66	21.19	24.72	28.25	31.78	35.32	38.85	42.38
0.045	3.97	7.95	11.92	15.89	19.87	23.84	27.81	31.78	35.76	39.73	43.70	47.68
0.050	4.41	8.83	13.24	17.66	22.07	26.49	30.90	35.32	39.73	44.15	48.56	52.97
0.075	6.62	13.24	19.87	26.49	33.11	39.73	46.35	52.97	59.60	66.22	72.84	79.46
0.100	8.83	17.66	26.49	35.32	44.15	52.97	61.80	70.63	79.46	88.29	97.12	105.95

*In actuality, the Francis turbine requires at least 0.5 m^3/s of flow, which we probably do not have

Table 6. Estimated Power Output (kW) from a Kaplan Turbine

	Net Head											
Flow Rate (m^3/s)	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**
0.0005	0.04	0.09	0.13	0.18	0.22	0.26	0.31	0.35	0.40	0.44	0.49	0.53
0.001	0.09	0.18	0.26	0.35	0.44	0.53	0.62	0.71	0.79	0.88	0.97	1.06
0.002	0.18	0.35	0.53	0.71	0.88	1.06	1.24	1.41	1.59	1.77	1.94	2.12
0.003	0.26	0.53	0.79	1.06	1.32	1.59	1.85	2.12	2.38	2.65	2.91	3.18
0.004	0.35	0.71	1.06	1.41	1.77	2.12	2.47	2.83	3.18	3.53	3.88	4.24
0.006	0.53	1.06	1.59	2.12	2.65	3.18	3.71	4.24	4.77	5.30	5.83	6.36
0.008	0.71	1.41	2.12	2.83	3.53	4.24	4.94	5.65	6.36	7.06	7.77	8.48
0.010	0.88	1.77	2.65	3.53	4.41	5.30	6.18	7.06	7.95	8.83	9.71	10.59
0.012	1.06	2.12	3.18	4.24	5.30	6.36	7.42	8.48	9.54	10.59	11.65	12.71
0.014	1.24	2.47	3.71	4.94	6.18	7.42	8.65	9.89	11.12	12.36	13.60	14.83
0.016	1.41	2.83	4.24	5.65	7.06	8.48	9.89	11.30	12.71	14.13	15.54	16.95
0.018	1.59	3.18	4.77	6.36	7.95	9.54	11.12	12.71	14.30	15.89	17.48	19.07
0.020	1.77	3.53	5.30	7.06	8.83	10.59	12.36	14.13	15.89	17.66	19.42	21.19
0.025	2.21	4.41	6.62	8.83	11.04	13.24	15.45	17.66	19.87	22.07	24.28	26.49
0.030	2.65	5.30	7.95	10.59	13.24	15.89	18.54	21.19	23.84	26.49	29.14	31.78
0.035	3.09	6.18	9.27	12.36	15.45	18.54	21.63	24.72	27.81	30.90	33.99	37.08
0.040	3.53	7.06	10.59	14.13	17.66	21.19	24.72	28.25	31.78	35.32	38.85	42.38
0.045	3.97	7.95	11.92	15.89	19.87	23.84	27.81	31.78	35.76	39.73	43.70	47.68
0.050	4.41	8.83	13.24	17.66	22.07	26.49	30.90	35.32	39.73	44.15	48.56	52.97
0.075	6.62	13.24	19.87	26.49	33.11	39.73	46.35	52.97	59.60	66.22	72.84	79.46
0.100	8.83	17.66	26.49	35.32	44.15	52.97	61.80	70.63	79.46	88.29	97.12	105.95

*Kaplan turbine requires net head less than 50 m

From these results, it becomes obvious that the Francis and Kaplan turbines are not the best choices based on our conditions. Therefore, we will consider only the Pelton, Banki-Mitchell, and Turgo in our further calculations.

These ideal combinations act as a starting point for choosing a location for the micro hydro system. However, the difference between net head and gross head must be accounted for. Whereas the net head is defined as the total energy head, the gross head is the physical difference in elevation between the top and the bottom of the system. The gross head usually must be larger than the net head, due to frictional losses. Therefore, the next step is to calculate the gross head for each of our ideal situations so we use the correct physical location. We can do this using a mechanical energy balance and the Bernoulli equation about the penstock. In order to complete the balance, a few more details about our system must be known.

Pipe Material and Diameter:

According to EnergyBible.com, the most common piping used in micro hydro systems is 4 inch PVC pipe or 2 inch flexible polyethylene pipe. Both materials have very low roughness, which is a definite advantage given the length of our penstock. PVC pipe offers the advantage of being sturdier and less easily damaged. However, it would also be harder to work with given the length of our system and the supports it would require. Flexible polyethylene pipe would be much easier to work with but could also be more easily damaged. We will consider both of these possible materials in each calculation. Because we will be receiving supplies in metric units, we will use standard metric sizes close to these values. Each calculation will be done with both 50 mm and 110 mm pipe diameters. In addition, a 100 mm long, 50 mm diameter nozzle will be used on the end of the wider penstock (110 mm).

Pipe Length:

We gained access to a topographical map of the area, seen below as *Figure 2*.

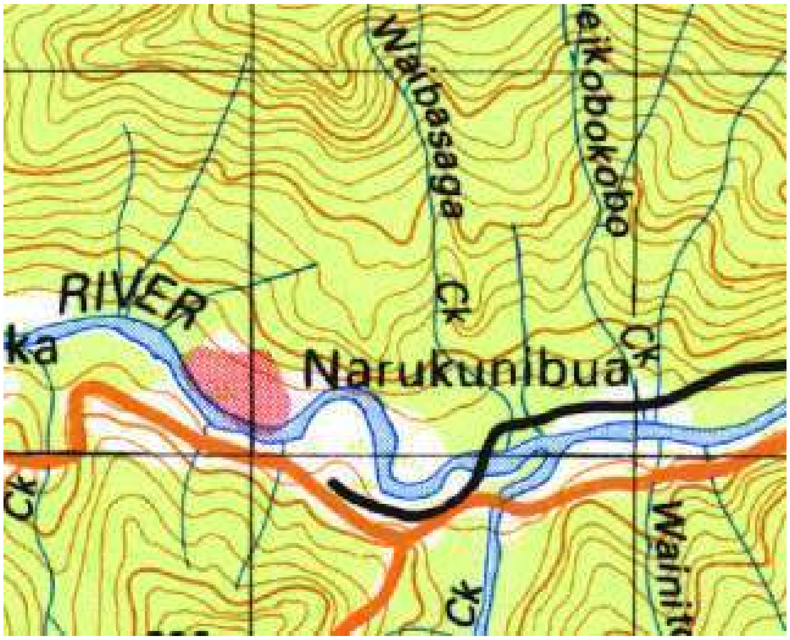


Figure 2. Topographical map of Narukunibua

The shaded red area is the location of the village. The three blue lines to the left are the possible streams that we will utilize for the system. The center stream appears to have the highest potential, since it starts at a higher elevation and may therefore have the highest flow rate. It also offers the highest possible gross head, 320 m. For this reason, it is the stream we will consider in our calculations. However, it is possible for use to change to one of the other streams once we have personally surveyed the location. Using this map, we can estimate the elevation change as well as the change in horizontal distance. Each brown line represents an elevation change of 20 m, while each of the black squares is 1000 m x 1000 m. We will use these two values to estimate a pipe length, increasing it slightly to account for bends and connectors in the pipes.

Additional Assumptions:

- Since both ends of the pipe are open the atmosphere, there should be no change in pressure.
- Using conservation of mass, the incoming flow rate must equal the exiting flow rate; therefore, the velocity will not change inside penstock since the diameter is constant
- Mass and therefore volume (water is incompressible at these conditions) is conserved about the nozzle, so the velocity will change to account for the change in diameter
- Assume both the pipe and the nozzle are made of smooth material (i.e. PVC)

Calculations:

Calculations will be performed for both pipe diameters (50 mm and 110 mm) for each of the bolded ideal combinations found on the previous charts (two per turbine).

Bernoulli Equation: $\frac{V_i^2}{2g}+\frac{P_i}{\rho}+h_i=\frac{V_f^2}{2g}+\frac{P_f}{\rho}+h_f+h_l=H_n$

where:

V = velocity [m/s]
P = pressure [Pa]
ρ = density of water [kg/m^3]
h_i = gross head [m]
h_l = head losses due to friction [m]
H_n = net head [m]

In our situation, the two pressure terms cancel out since both ends are open to atmospheric pressure, and h_2 is set equal to zero height. The equation then reduces to:

$$\frac{V_i^2}{2g}+h_i=\frac{V_f^2}{2g}+h_l=H_n$$

To find the initial velocity:

$$V_i = \frac{Q}{\frac{1}{4}\pi D^2}$$

To find the final velocity exiting the nozzle:

$$Q_i = Q_f = V_i * A_i = V_f * A_f$$

To find the head loss due to friction:

$$h_l = f\frac{L}{D}\frac{V^2}{2g}$$

An example of the above process will be shown in detail below; the rest will be displayed in *Tables* 7 & 8.

For the 15.89 kW of power produced by the Pelton wheel at 60 m net head and 0.03 m^3/s flow rate, using a 110 mm pipe diameter:

$$\frac{V_i^2}{2g} + \frac{P_i}{\rho} + h_i = \frac{V_f^2}{2g} + \frac{P_f}{\rho} + h_f + h_l = H_n$$

$$\frac{V_i^2}{2g} + h_i = \frac{V_f^2}{2g} + h_{l,p} + h_{l,n} = H_n$$

where:

$h_{l,p}$ = head loss through the pipe
$h_{l,n}$ = head loss through the nozzle

$$V_i = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.03}{\frac{1}{4}\pi * 0.11^2}$$ = 3.157 m/s

$Q_i = Q_f = 0.03 = V_f * \frac{1}{4}\pi * 0.05^2$, V_f = 15.28 m/s

From estimation on the map, L_{eq} ~ 185 m

Looking at the Levenspiel graph of friction factor vs. Reynolds number vs. relative roughness, assuming our pipe can be called “smooth,” the friction factor *f* = 0.002

$$\frac{V_i^2}{2g} + h_i = \frac{V_f^2}{2g} + h_{l,p} + h_{l,n} = H_n$$

$$\frac{3.157^2}{2g}+h_i=\frac{15.28^2}{2g}+\frac{0.002*185*3.157^2}{0.11*2g}+\frac{0.002*0.10*15.28^2}{0.05*2g}=60$$

h_i = 73.1 m

This process will now be repeated for a 50 mm pipe diameter. In this case, no nozzle is needed, so the equation will have a slightly different form (the two velocities are equal, so they cancel out, as does the friction loss through the nozzle).

$$V=\frac{Q}{\frac{1}{4}\pi D^2}=\frac{0.03}{\frac{1}{4}\pi*0.05^2} = 15.28 \text{ m/s}$$

$$h_i-h_{l,p}=H_n$$

$h_i-\frac{0.002*185*15.28^2}{0.05*2g}=60$, **$h_i$ = 148.0 m**

As one can see through this calculation, the smaller flexible polyethylene pipe may not be reasonable, as its faster velocity increases the friction losses enormously, creating a much higher gross head.

These calculations are applied to the following situations, and the calculated gross heads are displayed in *Tables 7 & 8* below. To see the complete calculations, see the Appendix.

Table 7. Gross head needed with 50 mm penstock

		Net Head [m]					
Turbine Type	**Flow Rate [m^3/s]**	**30**	**50**	**60**	**70**	**100**	**120**
Pelton	0.014						166.6
Turgo	0.018					160.0	
Banki-Mitchell	0.025				141.1		
Pelton	0.030			148.0			
Turgo	0.035		153.7				
Banki-Mitchell	0.060	201.4					

Table 8. Gross head needed with 110 mm penstock + 50 mm nozzle

		Net Head [m]					
Turbine Type	**Flow Rate [m^3/s]**	**30**	**50**	**60**	**70**	**100**	**120**
Pelton	0.014						123.4
Turgo	0.018					105.3	
Banki-Mitchell	0.025				79.3		
Pelton	0.030			73.1			
Turgo	0.035		67.6				
Banki-Mitchell	0.060	80.8					

Analysis of Results

Our initial calculations were very useful in the process of determining the ideal turbine and penstock combination for the micro hydro system. By combining these values with the advantages and disadvantages listed earlier, we can easily eliminate the Francis and Kaplan turbines as contenders. The Francis turbine requires a high flow rate, greater than 0.5 m^3/s, which we will probably not have in our situation. The Kaplan turbine does not have this explicit flow requirement, but it requires a low head. In order to produce the same amount of energy with a low head, there must again be a high flow rate. In addition, each of these turbines is generally more expensive to design and maintain. They are better suited to large power systems rather than our small system, so we can purge them from our list of options.

The three remaining turbines are all relatively similar in their characteristics, so we will probably wait until the area is personally surveyed before deciding between them.

What is obvious from our calculations is that a high head, low flow system is probably the optimal situation. There are the lowest frictional losses with this combination given the lower velocity. In addition, we are unsure how large our flow rate will be, so it is safer to assume a low flow rate. We know that we have up to 320 m of gross head available, so it is safer to make the assumption of a low flow, high head system.

The other conclusion from our calculations is that the 110 mm PVC pipe with a nozzle is a much more efficient system to use than the 50 mm flexible polyethylene pipe. Because the diameter is so much smaller in 50 mm penstock, the frictional losses are greatly increased. This causes our gross head to be much larger than it would need to be with the 110 mm pipe. However, the flexible pipe may prove to be much easier to work with, so increasing the elevation may be worth the sacrifice.

Concluding Remarks

Our final recommendation would be to use an impulse turbine, either the Pelton wheel, Banki-Mitchell turbine, or Turgo turbine. We suggest using a high head, low flow system with 110 mm PVC pipe to reduce frictional losses through the penstock. However, we have identified a varied collection of possible combinations that would produce the needed energy output. It seems that a micro hydro system could work very well in this environment, given the available elevation and rainfall.

References

Calculation of hydro power. Retrieved November 22, 2008, from Renewable Energy UK Web site: http://www.reuk.co.uk/Calculation-of-Hydro-Power.htm. (2008)

European Small Hydropower Association. *Guide on How to Develop a Small Hydropower Plant*. (2004).

Piping for micro hydro systems. Retrieved November 22, 2008, from Energy Bible Web site: http://www.energybible.com/water_energy/piping.html. (2008)

Appendix A

Here are the calculations for the values found in *Tables 7 & 8*

Pelton Wheel:

The calculations for *60 m net head, 0.03 m³/s flow* can be seen on pages 14-15.

120 m net head, 0.014 m³/s flow:

For 50 mm penstock:

$$V = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.014}{\frac{1}{4}\pi * 0.05^2} \text{ = 7.130 m/s}$$

$$h_i - h_{l,p} = H_n$$

$$h_i - \frac{0.002 * 450 * 7.130^2}{0.05 * 2g} = 120 \text{ , } \mathbf{h_i = 166.6\ m}$$

For 110 mm penstock:

$$V_i = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.014}{\frac{1}{4}\pi * 0.11^2} \text{ = 1.473 m/s}$$

$$Q_i = Q_f = 0.014 = V_f * \frac{1}{4}\pi * 0.05^2 \text{ , } V_f \text{ = 7.130 m/s}$$

$$\frac{V_i^{\ 2}}{2g} + h_i = \frac{V_f^{\ 2}}{2g} + h_{l,p} + h_{l,n} = H_n$$

$$\frac{1.473^2}{2g} + h_i = \frac{7.130^2}{2g} + \frac{0.002 * 450 * 1.473^2}{0.11 * 2g} + \frac{0.002 * 0.10 * 7.130^2}{0.05 * 2g} = 120 \text{ , } \mathbf{h_i = 123.4\ m}$$

Banki-Mitchell Turbine:

30 m net head, 0.060 m³/s flow:

For 50 mm penstock:

$$V = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.060}{\frac{1}{4}\pi * 0.05^2}$$ = 30.56 m/s

$$h_i - h_{l,p} = H_n$$

$$h_i - \frac{0.002 * 90 * 30.56^2}{0.05 * 2g} = 30$$, **h_i = 201.4 m**

For 110 mm penstock:

$$V_i = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.060}{\frac{1}{4}\pi * 0.11^2}$$ = 6.314 m/s

$$Q_i = Q_f = 0.06 = V_f * \frac{1}{4}\pi * 0.05^2$$, V_f = 30.56 m/s

$$\frac{V_i^2}{2g} + h_i = \frac{V_f^2}{2g} + h_{l,p} + h_{l,n} = H_n$$

$$\frac{6.314^2}{2g} + h_i = \frac{30.56^2}{2g} + \frac{0.002 * 90 * 6.314^2}{0.11 * 2g} + \frac{0.002 * 0.10 * 30.56^2}{0.05 * 2g} = 30$$, **h_i = 80.8 m**

70 m net head, 0.025 m^3/s flow:

For 50 mm penstock:

$$V = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.025}{\frac{1}{4}\pi * 0.05^2}$$ = 12.73 m/s

$$h_i - h_{l,p} = H_n$$

$$h_i - \frac{0.002 * 215 * 12.73^2}{0.05 * 2g} = 70$$, **h_i = 141.1 m**

For 110 mm penstock:

$$V_i = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.025}{\frac{1}{4}\pi * 0.11^2}$$ = 2.63 m/s

$$Q_i = Q_f = 0.025 = V_f * \frac{1}{4}\pi * 0.05^2$$, V_f = 12.73 m/s

$$\frac{V_i^2}{2g} + h_i = \frac{V_f^{\,2}}{2g} + h_{l,p} + h_{l,n} = H_n$$

$$\frac{2.63^2}{2g} + h_i = \frac{12.73^2}{2g} + \frac{0.002 * 215 * 2.63^2}{0.11 * 2g} + \frac{0.002 * 0.10 * 12.73^2}{0.05 * 2g} = 70$$, **h_i = 79.3 m**

Turgo Turbine:

50 m net head, 0.035 m^3/s flow:

For 50 mm penstock:

$$V = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.035}{\frac{1}{4}\pi * 0.05^2}$$ = 17.83 m/s

$$h_i - h_{l,p} = H_n$$

$$h_i - \frac{0.002 * 160 * 17.83^2}{0.05 * 2g} = 50$$, **h_i = 153.7 m**

For 110 mm penstock:

$$V_i = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.035}{\frac{1}{4}\pi * 0.11^2}$$ = 3.683 m/s

$$Q_i = Q_f = 0.035 = V_f * \frac{1}{4}\pi * 0.05^2$$, V_f = 17.83 m/s

$$\frac{V_i^2}{2g} + h_i = \frac{V_f^{\,2}}{2g} + h_{l,p} + h_{l,n} = H_n$$

$$\frac{3.683^2}{2g} + h_i = \frac{17.83^2}{2g} + \frac{0.002 * 160 * 3.683^2}{0.11 * 2g} + \frac{0.002 * 0.10 * 17.83^2}{0.05 * 2g} = 50$$, **h_i = 67.6 m**

100 m net head, 0.018 m^3/s flow:

For 50 mm penstock:

$$V = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.018}{\frac{1}{4}\pi * 0.05^2}$$ = 9.167 m/s

$$h_i - h_{l,p} = H_n$$

$$h_i - \frac{0.002 * 350 * 9.167^2}{0.05 * 2g} = 100$$, **h_i = 160.0 m**

For 110 mm penstock:

$$V_i = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.018}{\frac{1}{4}\pi * 0.11^2}$$ = 1.894 m/s

$$Q_i = Q_f = 0.018 = V_f * \frac{1}{4}\pi * 0.05^2$$, V_f = 9.167 m/s

$$\frac{V_i^{\,2}}{2g} + h_i = \frac{V_f^{\,2}}{2g} + h_{l,p} + h_{l,n} = H_n$$

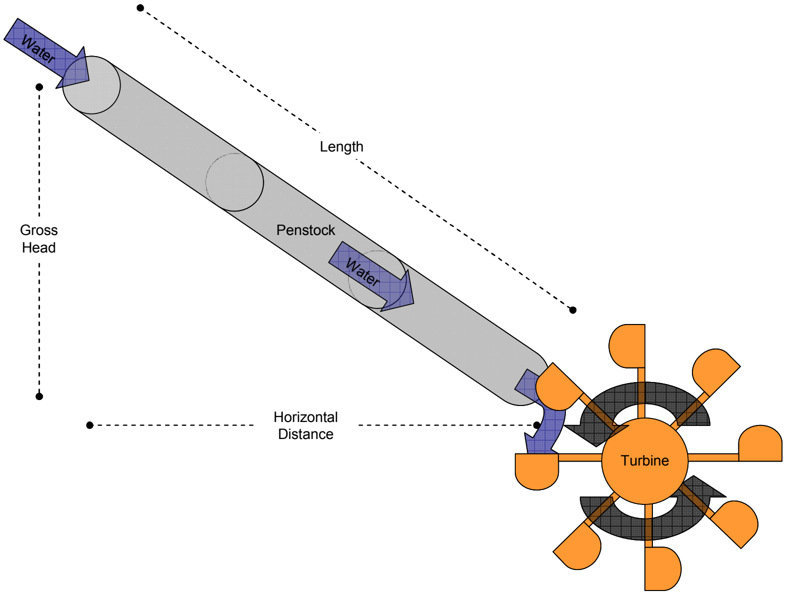
$$\frac{1.894^2}{2g} + h_i = \frac{9.167^2}{2g} + \frac{0.002 * 350 * 1.894^2}{0.11 * 2g} + \frac{0.002 * 0.10 * 9.167^2}{0.05 * 2g} = 100$$, **h_i = 105.3 m**

Appendix B

Example Class Problem

The following data describes a situation in which 15.89 kW of power is produced by the Pelton wheel. The net head (H_n) is 60 [m], the flow rate (Q) is 0.03 [m^3/s], the equivalent length (L_{eq}) is 185 [m], and the friction factor (*f*), assuming a smooth pipe, is 0.002.

a. Find the gross head needed using a 50 mm diameter penstock.
b. Find the gross head needed using a 110 mm diameter penstock with a 50 mm diameter nozzle (0.1 m in length) at the end of the penstock.
c. Which diameter seems to be the best choice for the system?



Solution:

a. To find the velocity through the penstock:

$$V = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.03}{\frac{1}{4}\pi * 0.05^2} = 15.28 \text{ m/s}$$

Starting with the mechanical energy balance in Bernoulli equation form:

$$\frac{V_i^{\ 2}}{2g} + \frac{P_i}{\rho} + h_i = \frac{V_f^{\ 2}}{2g} + \frac{P_f}{\rho} + h_f + h_l = H_n$$

We see that the initial and final pressures are the same, since both ends of the penstock are open to atmospheric pressure. Also, due to conservation of mass, the initial and final velocities are also the same. We set our h_f equal to zero. With these simplifications, the equation simply becomes:

$$h_i - h_{l,p} = H_n$$

The head loss due to friction can be found by:

$$h_l = f\frac{L}{D}\frac{V^2}{2g}$$

Plugging in the values:

$$h_i - \frac{0.002*185*15.28^2}{0.05*2g} = 60$$, ***h***

b. Use the process to find the initial velocity through the penstock (before the nozzle):

$$V_i = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.03}{\frac{1}{4}\pi*0.11^2}$$ = 3.157 m/s

To find the final velocity exiting the nozzle:

$$Q_i = Q_f = V_i * A_i = V_f * A_f$$

$$Q_i = Q_f = 0.03 = V_f * \frac{1}{4}\pi*0.05^2$$, V_f = 15.28 m/s

In the Bernoulli form of the mechanical energy balance, the initial and final velocities do not cancel out this time, since the velocity is increased after exiting the nozzle. The two pressure terms do still cancel, as well as the h_f term. In this case, we must account for frictional losses in both the penstock and the nozzle, abbreviated as $h_{l,p}$ and $h_{l,n}$.

$$\frac{V_i^2}{2g} + h_i = \frac{V_f^2}{2g} + h_{l,p} + h_{l,n} = H_n$$

$$\frac{3.157^2}{2g} + h_i = \frac{15.28^2}{2g} + \frac{0.002*185*3.157^2}{0.11*2g} + \frac{0.002*0.10*15.28^2}{0.05*2g} = 60$$

h_i = 73.1 m

c. The diameter/nozzle combination in part ‘b’ is probably a better choice for the system. There are much fewer losses due to friction with the larger penstock, since the velocity is not as fast. However, the same final velocity is attained with both systems due to the nozzle at the end of the larger penstock.

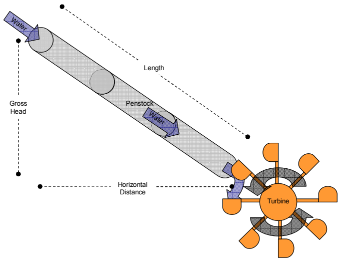
Fluid Flow in a Micro Hydro System

Example Problem

Victoria Johnson and Jenna Wilson
CHE 331 Special Project

Micro Hydro Example Problem

The following data describes a situation in which 15.89 kW of power is produced by the Pelton wheel. The net head (Hn) is 60 [m], the flow rate (Q) is 0.03 [m3/s], the equivalent length (Leq) is 185 [m], and the friction factor (*f*), assuming a smooth pipe, is 0.002.

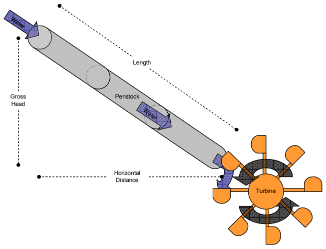


Micro Hydro Example Problem

a. Find the gross head needed using a 50 mm diameter penstock.

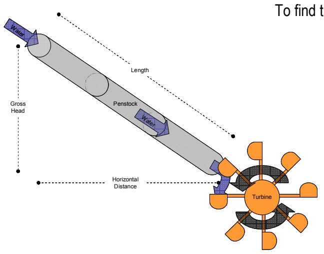
b. Find the gross head needed using a 110 mm diameter penstock with a 50mm diameter nozzle (0.1 m in length) at the end of the penstock.

c. Which diameter seems to be the best choice for the system?



Micro Hydro Example Problem

a. *Find the gross head needed using a 50 mm diameter penstock.*



To find the velocity through the penstock:

$$V = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.03}{\frac{1}{4}\pi * 0.05^2}$$

= 15.28 m/s

Micro Hydro Example Problem

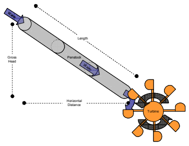
a. *Find the gross head needed using a 50 mm diameter penstock.*

Starting with the mechanical energy balance in Bernoulli equation form:

$$\frac{V_i^2}{2g} + \frac{P_i}{\rho} + h_i = \frac{V_f^2}{2g} + \frac{P_f}{\rho} + h_f + h_l = H_n$$

We can make the following simplifications:

- Pressure at both ends is atmospheric
- Initial velocity equals final velocity (conservation of mass)
- H_f is equal to zero



Micro Hydro Example Problem

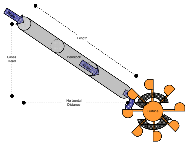
a. *Find the gross head needed using a 50 mm diameter penstock.*

With these simplifications, the equation takes the form:

$$h_i - h_{l,p} = H_n$$

The head loss due to friction can be found by:

$$h_l = f\frac{L}{D}\frac{V^2}{2g}$$



Micro Hydro Example Problem

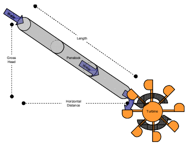
a. Find the gross head needed using a 50 mm diameter penstock.

Plugging in the values:

$$h_i - h_{l,p} = H_n$$

$$h_i - \frac{0.002*185*15.28^2}{0.05*2g} = 60$$

$$\boxed{h_i = 148.0 \text{ m}}$$

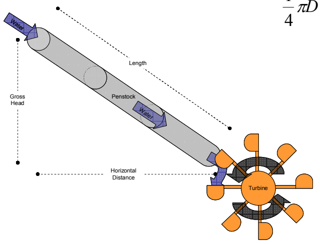


Micro Hydro Example Problem

b. Find the gross head needed using a 110 mm diameter penstock with a 50mm diameter nozzle (0.1 m in length) at the end of the penstock.

To find the initial velocity through the penstock (before the nozzle):

$$V_i = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{0.03}{\frac{1}{4}\pi * 0.11^2} \quad \mathbf{= 3.157\ m/s}$$



Micro Hydro Example Problem

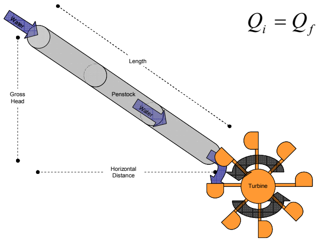
b. Find the gross head needed using a 110 mm diameter penstock with a 50mm diameter nozzle (0.1 m in length) at the end of the penstock.

To find the final velocity exiting the nozzle:

$$Q_i = Q_f = V_i * A_i = V_f * A_f$$

$$Q_i = Q_f = 0.03 = V_f * \frac{1}{4}\pi * 0.05^2$$

V_f = 15.28 m/s



Micro Hydro Example Problem

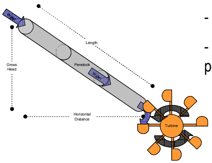
b. Find the gross head needed using a 110 mm diameter penstock with a 50mm diameter nozzle (0.1 m in length) at the end of the penstock.

Starting with the mechanical energy balance in Bernoulli equation form:

$$\frac{V_i^2}{2g} + \frac{P_i}{\rho} + h_i = \frac{V_f^2}{2g} + \frac{P_f}{\rho} + h_f + h_l = H_n$$

We can make the following simplifications:

- Pressure at both ends is atmospheric
- H_f is equal to zero
- Must account for frictional head losses through both the penstock and the nozzle



Micro Hydro Example Problem

b. Find the gross head needed using a 110 mm diameter penstock with a 50mm diameter nozzle (0.1 m in length) at the end of the penstock.

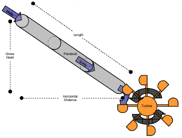
With these simplifications, the equation takes the form:

$$\frac{V_i^2}{2g} + h_i = \frac{V_f^2}{2g} + h_{l,p} + h_{l,n} = H_n$$

Plugging in the values:

$$\frac{3.157^2}{2g} + h_i = \frac{15.28^2}{2g} + \frac{0.002*185*3.157^2}{0.11*2g} + \frac{0.002*0.10*15.28^2}{0.05*2g} = 60$$

$$\boxed{h_i = 73.1 \text{ m}}$$



Micro Hydro Example Problem

c. Which diameter seems to be the best choice for the system?

The diameter/nozzle combination in part ‘b’ is probably a better choice for the system. There are much fewer losses due to friction with the larger penstock, since the velocity is not as fast. However, the same final velocity is attained with both systems due to the nozzle at the end of the larger penstock.